# CIRCLE PROPERTIES

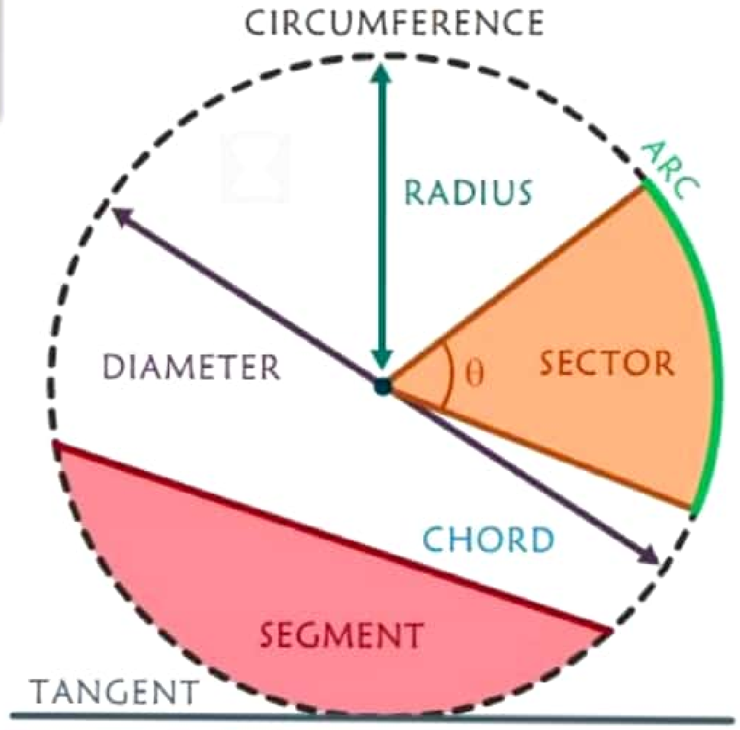
**Part I**

**CIRCUMFERENCE**
Length of the outer edge of a circle

**DIAMETER**
The distance from edge to edge passing through the center

**CHORD**
A straight line joining any two points on the circumference

**SEGMENT**
The area inside a circle enclosed by an arc and a chord




**RADIUS**
The distance from the center to the edge; half the diameter.

**SECTOR**
The area enclosed by an arc and two radii

**ARC**
A part of the circumference

**TANGENT**
A straight line that touches a circle at two coincident points

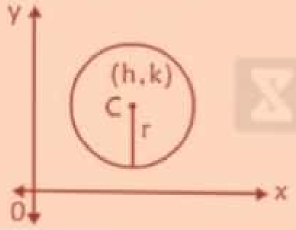
| Area | Area of Sector | Circumference | Length of Arc |
|---|---|---|---|
| $\text{Area} = \pi r^2$ | $\text{Area of Sector} = \pi r^2 \cdot \frac{\theta}{360^\circ}$ | $\text{Circumference} = 2\pi r$ | $\text{Length of Arc} = 2\pi r \cdot \frac{\theta}{360^\circ}$ |

$\theta \rightarrow$ in degrees

## STANDARD EQUATION OF THE CIRCLE

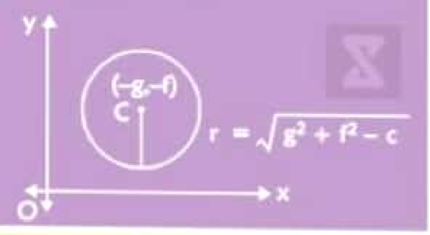
**1. Central Form:**

$(x-h)^2 + (y-k)^2 = r^2$



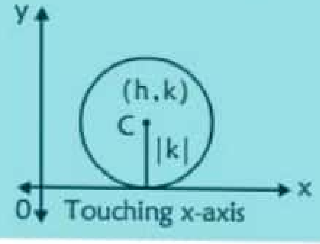

**2. General equation of circle:**

$x^2 + y^2 + 2gx + 2fy + c = 0$



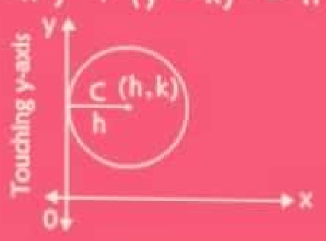

**3. When circle touches x-axis**

$(x-h)^2 + (y-k)^2 = k^2$



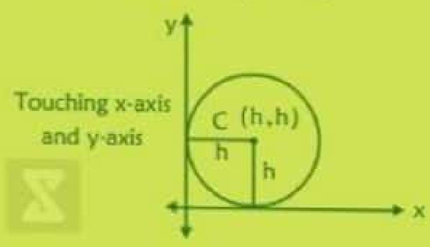

**4. When circle touches y-axis**

$(x-h^2) + (y-k)^2 = h^2$



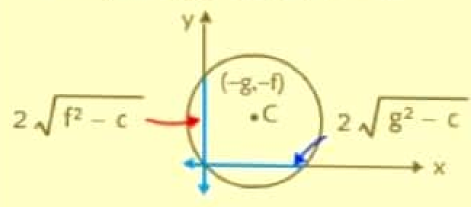

**5. When circle touches both the axis**

$(x-h^2) + (y-h)^2 = h^2$



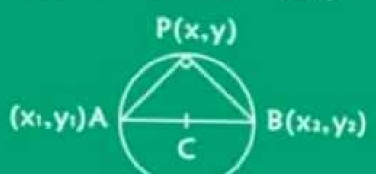

**6. Intercepts cut by the circle on axes:**

$x^2 + y^2 + 2gx + 2fy + c = 0$




**7. Diametrical form of circle:**

$(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$




**8. The parametric forms of the circle:**

| Circle | Parametric forms |
|---|---|
| $x^2 + y^2 = r^2$ | $x = r\cos\theta,\ y = r\sin\theta$ |
| $(x-h)^2 + (y-k)^2 = r^2$ | $x = h + r\cos\theta,\ y = k + r\sin\theta$ |

where $\theta$ is the parameter $\theta \in [0, 2\pi)$

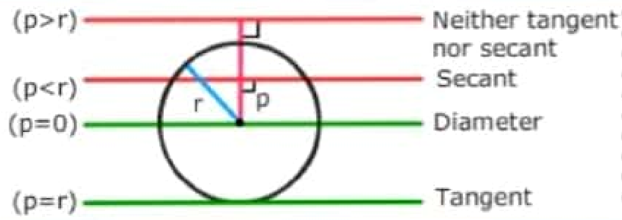
# CIRCLE'S TANGENTS

**Part II**

**Condition of Tangency :**

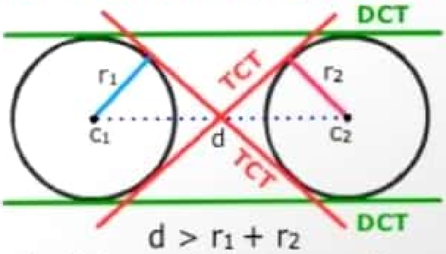
p = perpendicular distance from center to line

r = radius to circle




## Common tangents of two circles

### 1 If two circle are separated

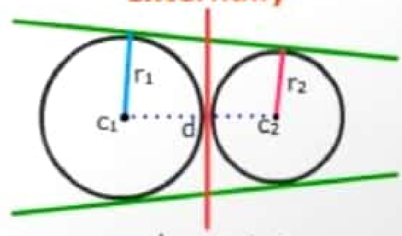



$d > r_1 + r_2$

d=distance between centers

$\Rightarrow$ 4 common tangents (2DCT,2TCT)

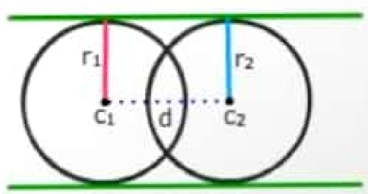
### 2 If two circles touch externally




$d = r_1 + r_2$

$\Rightarrow$ 3 common tangents (2DCT,1TCT)

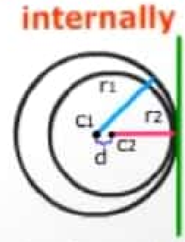
### 3 If two circles intersect each other




$|r_1 - r_2| < d < r_1 + r_2$

$\Rightarrow$ 2 common tangents (2DCT)

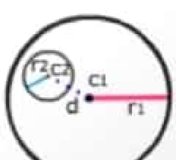
### 4 If two circles touch internally




$d = |r_1 - r_2|$

$\Rightarrow$ 1 common tangent (1DCT)

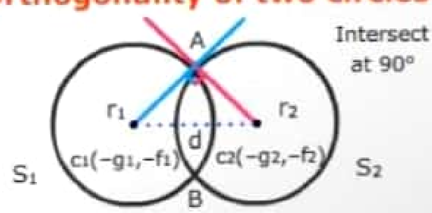
### 5 If one circles is completely contained in another circle




$d < |r_1 - r_2|$

$\Rightarrow$ No common tangent

### 6 Orthogonality of two circles




$x^2+y^2+2g_1x+2f_1y+d_1=0$

$x^2+y^2+2g_2x+2f_2y+d_2=0$

**Condition : $2g_1g_2 + 2f_1f_2 = d_1+d_2$**

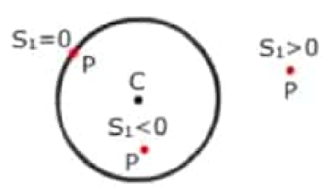
*** DCT = Direct Common Tangent, TCT = Transverse Common Tangent**

**Position of a point $P(x_1,y_1)$ w.r.t. circle :**

$x^2+y^2+2gx+2fy+c = 0$

deponds on

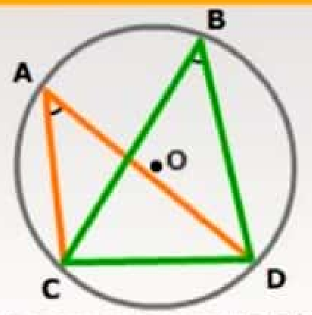
$S_1 = x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c$



# CIRCLE THEOREMS

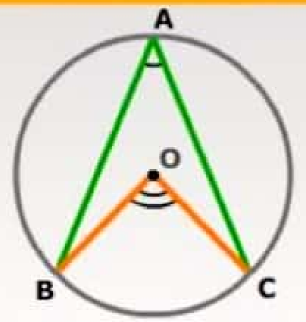
**Part III**

## 1 ∠CAD = ∠CBD



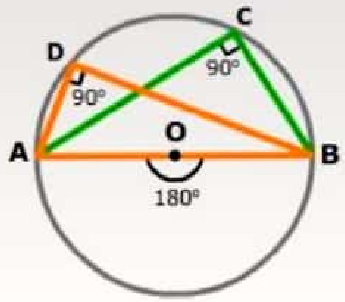

Angle in the same segment and standing on the same chord are always equal.

## 2 ∠BOC = 2∠BAC



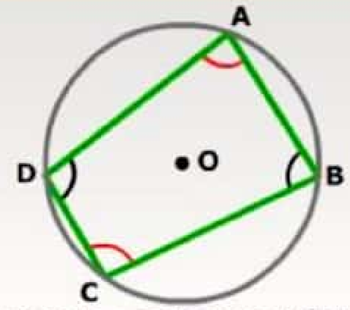

The angle at the centre of a circle is twice the angle at the circumference.

## 3 ∠BCA = ∠BDA = 90°



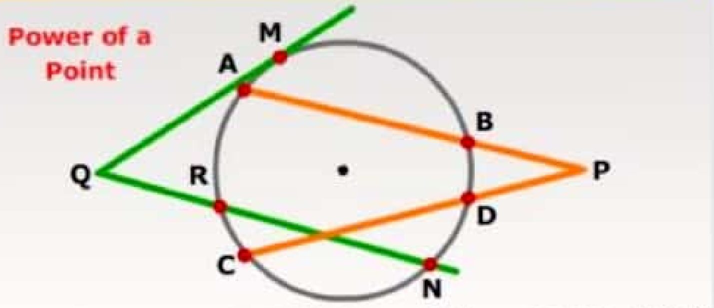

The angle in a semi-circle is always 90°.

## 4 ∠B + ∠D = 180°, ∠A + ∠C = 180°



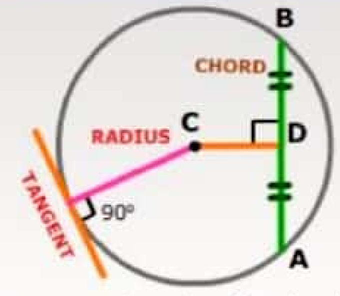

ABCD is a cyclic Quadrilateral, Diagonally opposite angles add up to 180°.

## 5 QR.QN = (QM)², PA.PB = PC.PD



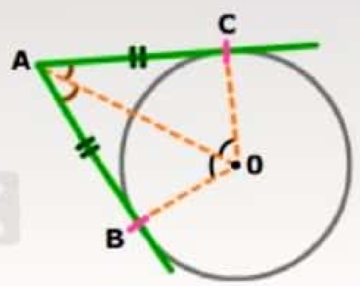

If QRN is a secant & QM is tangent then QR.QN=(QM)²
If PBA & PDC are secant to circle then PA.PB=PC.PD

## 6 CD ⊥ AB, BD = DA, ∠BDC = 90°



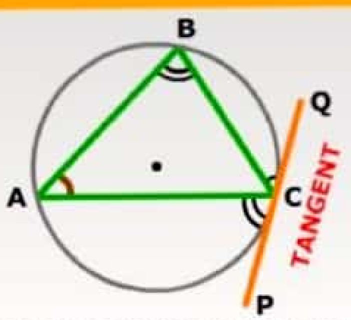

Angle between tangent and radius is always 90°.
Perpendicular bisector of any chord, pass through center.

## 7 AB = AC, ∠AOB = ∠AOC




Tangents from a common point (A) to a circle are always equal.

## 8 ∠PCA = ∠ABC, ∠QCB = ∠BAC




The angle between the tangent and the side of the triangle is equal to the interior opposite angle.